

## Computational Complexity in Analysis

### SoSe 2015, Exercise Sheet #1

#### EXERCISE 1:

- a) Prove that there exists some  $V \in \mathcal{P}$  such that, w.r.t. inputs encoded in binary, the following decision problem is  $\mathcal{NP}$ -complete:

$$\{x \in \mathbb{N} : \exists y \leq x : \langle x, y \rangle \in V\}$$

- b) Prove that there exists some  $V \in \mathcal{P}$  such that, w.r.t. inputs encoded in binary, the following function problem is  $\#\mathcal{P}$ -complete:

$$\mathbb{N} \ni x \mapsto \#\{y \leq x : \langle x, y \rangle \in V\}$$

- c) Prove  $\#\mathcal{P} \subseteq \text{FPSPACE}$ , the class of function computable in polynomial space.

#### EXERCISE 2:

- a) Prove that  $\text{LOGSPACE}$  is closed under composition.
- b) Prove that  $\mathcal{P}^{\mathcal{P}} = \mathcal{P}$ .
- c) Prove that  $\text{PSPACE}^{\text{PSPACE}} = \text{PSPACE}$ .