

## Computational Complexity in Analysis

### SoSe 2015, Exercise Sheet #3

#### EXERCISE 5:

- a) Prove that the following decision problem can be solved in exponential, but not in polynomial, time:

$$\{1^k 0 \langle \mathcal{M} \rangle \mid k \in \mathbb{N}, \text{ Turing machine } \mathcal{M} \text{ rejects input } 1^k 0 \langle \mathcal{M} \rangle, \\ \text{or takes more than } k \cdot |\langle \mathcal{M} \rangle|^k \text{ steps to accept it}\}$$

- b)  $L \subseteq \mathbb{N}$  is decidable in polynomial time iff the real number  $\sum_{x \in L} 4^{-x}$  is polytime computable.

#### EXERCISE 6:

- a) Prove  $\exp : [-2^k; k] \rightarrow \mathbb{R}$  computable within parameterized polynomial time  $(n+k)^{O(1)}$ .  
b) Prove that  $\exp : \mathbb{R} \rightarrow \mathbb{R}$  is not computable within time  $t(n)$  for any  $t : \mathbb{N} \rightarrow \mathbb{N}$ .  
c) Prove that the following function  $h : [0; 1] \rightarrow [0; 1]$  is computable in exponential, but not in polynomial time:

$$h(0) = 0, \quad h(t) = 1/\ln(e/t)$$

