Martin Ziegler
Holger Thies

Issued on 2015-05-19

Computational Complexity in Analysis

SoSe 2015, Exercise Sheet #6

EXERCISE 11:
Let $f : [0;1] \to [0;1]$ be polynomial-time computable and $F :\subseteq \mathbb{Z} \times \mathbb{N} \to \mathbb{Z}$ as in Exercise 10a), that is, polynomial-time computable satisfying

$$\left| f(a/2^n) - F(a,2^n)/2^n \right| \leq 1/2^n \quad \text{whenever} \quad 0 \leq a \leq 2^n \ . \tag{1}$$

a) Prove that the following counting problem belongs to $\#\mathsf{P}$:

$$\mathtt{1}^m\,\mathtt{0}\,\mathrm{bin}(c) \;\mapsto\; \mathrm{Card}\left\{(a,b) \in \mathbb{Z}^2 : 0 \leq a \leq c :\; F(a,2^m) \geq b\right\}$$

b) Conclude that $\mathsf{FP} = \#\mathsf{P}$ implies polynomial-time computability of the function $\int f : [0;1] \ni x \mapsto \int_0^x f(t)\,dt$.

c) Prove that the following counting problem belongs to $\#\mathsf{P}_1$:

$$\mathtt{1}^m \;\mapsto\; \mathrm{Card}\left\{(a,b) \in \mathbb{Z}^2 : 0 \leq a \leq 2^m :\; F(a,2^m) \geq b\right\}$$

d) Conclude that $\mathsf{FP}_1 = \#\mathsf{P}_1$ implies polynomial-time computability of the real number $\int_0^1 f(t)\,dt$.