Martin Ziegler
Holger Thies

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Computational Complexity in Analysis

SoSe 2015, Exercise Sheet #9

The lecture has proven the following as equivalent:

i) $\mathsf{FP} = \#\mathsf{P}$

ii) For every polynomial-time computable $h : [0;1] \to \mathbb{R}$, the function $\int h : [0;1] \to \mathbb{R}$ with $x \mapsto \int_0^x h(t)\,dt$ is again polynomial-time computable.

iii) For every smooth (i.e. C^∞) polynomial-time computable $h : [0;1] \to \mathbb{R}$ with support $\mathrm{supp}(f) \subseteq [1/4;3/4]$, $\int h$ is again polynomial-time computable.

Recall Poisson's partial differential equation

$$\Delta u = f \text{ in } \Omega, \quad u|_{\partial\Omega} = g \tag{1}$$

for some fixed bounded, open, and connected $\Omega \subseteq \mathbb{R}^d$ with boundary $\partial\Omega$ and given $f : \Omega \to \mathbb{R}$ and $g : \partial\Omega \to \mathbb{R}$, where $\Delta u(x_1,\ldots,x_d) = \partial^2_{x_1} u(\vec{x}) + \partial^2_{x_2} u(\vec{x}) + \ldots + \partial^2_{x_d} u(\vec{x})$.

EXERCISE 15:

a) Let $\Omega_d = \{\vec{x} : \|\vec{x}\|_2 < 1\} \subseteq \mathbb{R}^d$ and suppose $u : \Omega_d \to \mathbb{R}$ is radially symmetric, i.e., $u(\vec{x}) = v(\|\vec{x}\|)$ for some $v : [0;1) \to \mathbb{R}$. Show that $\Delta v(r) = \left(r^{d-1} \cdot v'\right)' / r^{1-d}$ for $r > 0$ and $d = 1, 2, 3, \ldots$
Hint: Spherical coordinates.

b) Construct a polynomial-time computable smooth $f : \Omega_d \to \mathbb{R}$ such that the solution u to Equation (1) is not polynomial-time computable unless FP=$\#\mathsf{P}$.