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CS700

Fall 2016, Assignment #1

PROBLEM 1 (2+2+3+3)**:**
Recall that decision problem $X \subseteq \{0,1\}^*$ is called *reducible* to $Y \subseteq \{0,1\}^*$ (written $X \preccurlyeq Y$) if there exists a total computable function $f : \{0,1\}^* \to \{0,1\}^*$ such that, for all $\vec{x} \in \{0,1\}^*$, it holds: $\vec{x} \in X \Leftrightarrow f(\vec{x}) \in Y$.

a) Prove $T \preccurlyeq E$.

b) Prove $E \preccurlyeq T$.

c) Prove $\bar{T} \preccurlyeq H^H$.

d) Prove $H^H \preccurlyeq \bar{T}$.

Here we recall the Totality problem T and consider the following problems:

E) Given two (finite binary strings encoding) Turing machines/algorithms $\mathcal{A}$ and $\mathcal{B}$, are they equivalent in the sense that, for every $\vec{x} \in \{0,1\}^*$, $\mathcal{A}$ on input $\vec{x}$ eventually terminates iff $\mathcal{B}$ on input $\vec{x}$ does (although not necessarily after the same number of steps)?

T) Given a Turing machine/algorithm $\mathcal{A}$, does it terminate on *all* possible inputs $\vec{x}$?

H^H) Given a Turing machine/algorithm $\mathcal{A}^H$ with oracle access to the Halting problem H and input $\vec{x}$, does it terminate?

Solution to d): $\langle \mathcal{A}^H \rangle \mapsto \langle \mathcal{B}_{\mathcal{A}} \rangle$, where $\mathcal{B}_{\mathcal{A}}(t)$

- simulates $\mathcal{A}^?$ and aborts if does not stop within $\leq t$ steps
- answering (correctly) positive any query "$q \in H$?" made by $\mathcal{A}$
  that succeeds when semi-deciding it for up to t steps
- while otherwise answering it (possibly incorrectly) to the negative.
- If successful up to here (i.e. $\mathcal{A}$'s simulation has stopped),
  keep simultaneously semi-deciding those queries "$q \in H$?" answered negatively.