Martin Ziegler
Jiwon Park

Issued on Nov.29, 2016
Solutions due: Dec.12, 2016

CS700

Fall 2016, Assignment #4

PROBLEM 5 (2+1+1+1+2P)**:**
Which of the following are compact metric spaces? Dis-/prove!

a) $\mathrm{Lip}_L([0;1],[0;1]) := \{f:[0;1]\to[0;1],\ |f(x)-f(x')|\leq L\cdot|x-x'|\}$
with $d_\infty(f,g) := \sup_{x\in[0;1]}|f(x)-g(x)|$

b) $\mathrm{Lip}_L([0;1]) = \{f:[0;1]\to\mathbb{R},\ |f(x)-f(x')|\leq L\cdot|x-x'|\}$ with d_∞

c) $\mathrm{Lip}([0;1],[0;1]) := \bigcup_{L>0}\mathrm{Lip}_L([0;1],[0;1])$ with d_∞

d) $\{f\in\mathrm{Lip}_1([0;1],[0;1])$ continuously differentiable$\}$ with d_∞

e) $\{f\in\mathrm{Lip}_1([0;1],[0;1])$ continuously differentiable$\}$ with $d_{\infty,\infty}(f,g):=d_\infty(f,g)+d_\infty(f',g')$

Recall that $f:[-1;1]\to\mathbb{R}$ is computable in time $t:\mathbb{N}\to\mathbb{N}$ if some Turing machine can convert any sequence $(a_m)\subseteq\mathbb{Z}$ satisfying $|x-a_m/2^m|\leq 2^{-m}$ for some $x\in[-1;1]$ to a sequence $(b_n)\subseteq\mathbb{Z}$ satisfying $|f(x)-b_n/2^n|\leq 2^{-n}$ such that b_n appears within $t(n)$ steps.

PROBLEM 6 (1+1+1+2+1+2P)**:**

a) Prove that $\exp_k:[0;k]\ni x\mapsto e^x$ is computable in time polynomial in $n+k$, $k\in\mathbb{N}$.

b) Prove that $\exp_k:[0;2^k]\ni x\mapsto e^x$ is not computable in time polynomial in $n+k$.

c) Prove that $f_k:[0;2^k]\ni x\mapsto x^2$ is computable in time polynomial in $n+k$.

d) Prove that $[1;\infty)\ni x\mapsto 1/x\in(0;1]$ is computable in time polynomial in n.

e) Let $f:[0;1]\to\mathbb{R}$ and $g:[-1;0]\to\mathbb{R}$ be computable in polynomial time with $f(0)=g(0)$. Prove that the joined function $[-1;1]\to\mathbb{R}$ is again computable in polynomial time.

f) Suppose $f:[0;1]\to\mathbb{R}$ is computable in polynomial time and twice continuously differentiable. Prove that its derivative $f':[0;1]\to\mathbb{R}$ is again computable in polynomial time.